

# DETECTION OF DAMAGE IN AXIAL (MEMBRANE) SYSTEMS

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## ABSTRACT

In a recent paper, two methods of damage identification ('Modified Damage-Index' and 'Change-in-Flexibility') were applied to detection of damage in an 8-DOF vibrating system. The goal of the work was to detect damage (reduction in stiffness of one or more of the elements) as well as to locate the particular damaged element(s). However, the investigation was limited to numerical simulations only. In this paper, a physical, spring-mass model of a similar, degenerate 8-DOF system (7 normal modes plus a rigid-body mode) was constructed. Experiments were then performed and the modal properties of the system were determined in "undamaged" and "damaged" states. Excitation was provided either by an impact hammer or by an electro-dynamic shaker. Damage was induced by replacing one of the springs with a spring of lower stiffness. The Modified 'Damage Index' method clearly isolated the location of damage for a variety of damage locations and levels of damage. The 'Change-in-Flexibility' method, however, was found to be less reliable. The ability of the method to locate damage depended strongly on location and the level of damage as well as the number of modes included.

## NOMENCLATURE

$a$	:Axial position along span
$AE$	:Axial rigidity
$EI$	:Flexural rigidity
$g_{ik}$	:Axial rigidity ratio for $i^{\text{th}}$ mode shape and $k^{\text{th}}$ region
$\ell$	:Span of structure
$n$	:Number of measured modes
$U$	:Strain energy
$u$	:In-plane displacement
$w$	:Transverse displacement
$x$	:Axial Coordinate
$\alpha_k$	:Damage index
$\delta_{ij}$	:Elements of change-in-flexibility matrix
$\omega_i$	: $i^{\text{th}}$ frequency of vibration
$[F]$	:Flexibility matrix
$\{\psi_i\}$	: $i^{\text{th}}$ mode shape
$[\Psi]$	:Mode shape matrix

$[\Delta F]$	:Change-in-flexibility matrix
$[\Omega]$	:Modal stiffness matrix
$\{\phi_i\}$	: $i^{\text{th}}$ mass-normalized mode shape
$[\Phi]$	:Mass-normalized mode shape matrix
$[ ]^T$	:Transpose
$[ ]^{-1}$	:Inverse
$( )^*$	:Property of damaged structure

## 1 INTRODUCTION

Numerous vibration-based damage identification methods have been reported for both the detection of damage in structures and/or the determination of the damaged location [1,2]. These methods can generally be partitioned into one of two classes: "Model-based" or "non-model based". By far the largest application of these vibration-based damage ID methods to date has been to structures, such as bridges, undergoing flexural vibrations. Damage detection in truss structures exhibiting bending, torsional and axial modes is discussed in [3,4].

In [5], two recently reported non-model based damage ID methods, which had been previously utilized for flexural vibrations only, were applied to axial-type vibrations of an 8-DOF linear spring-mass system. The goal of that work was to detect damage (as indicated by a reduction in stiffness of one or more of the springs) as well as to locate the damaged spring. The two damage detection methods utilized were both found to successfully locate the damaged spring(s) for a 10-percent reduction in element stiffness. However these results were based on numerical simulations only. No actual experimental data were used in that earlier study.

It is the purpose of this paper to report an evaluation of these two linear non-model-based damage ID methods when utilizing actual experimental data. First, the 8-DOF spring-mass experiments are described, followed by a brief summary of the damage ID methods for detecting and locating damage in axial (membrane), as opposed to flexural, systems. Then the methods are used with the forced (shaker and impact-hammer excitation) vibration experimental data in an attempt to locate damage. 'Damage' was introduced at a variety of locations and

with a variety of magnitudes by replacing selected springs with weaker counterparts.

## 2 EXPERIMENTS

An eight degree-of-freedom spring-mass system was designed and constructed to study the effectiveness of the two vibration-based damage identification techniques. The system is formed with eight translating masses connected by springs.

A schematic of the system is shown in Figure 1. Each mass is a disc of aluminum 25.4 mm thick and 76.2 mm in diameter with a center hole. The hole is lined with a Teflon bushing. There are small steel collars on each end of the discs. The masses all slide on a highly polished steel rod that supports the masses and constrains them to translate along the rod. The masses are fastened together with coil springs epoxied to the collars that are, in turn, bolted to the masses as shown in the Figures 1 and 2.

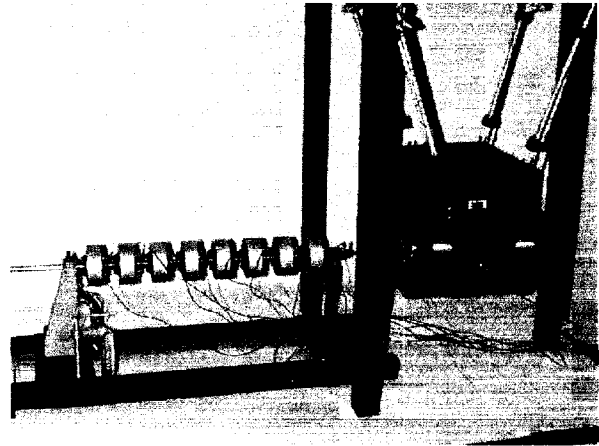


Figure 2: Eight degree-of-freedom system attached to electrodynamic shaker with accelerometers mounted on each mass.

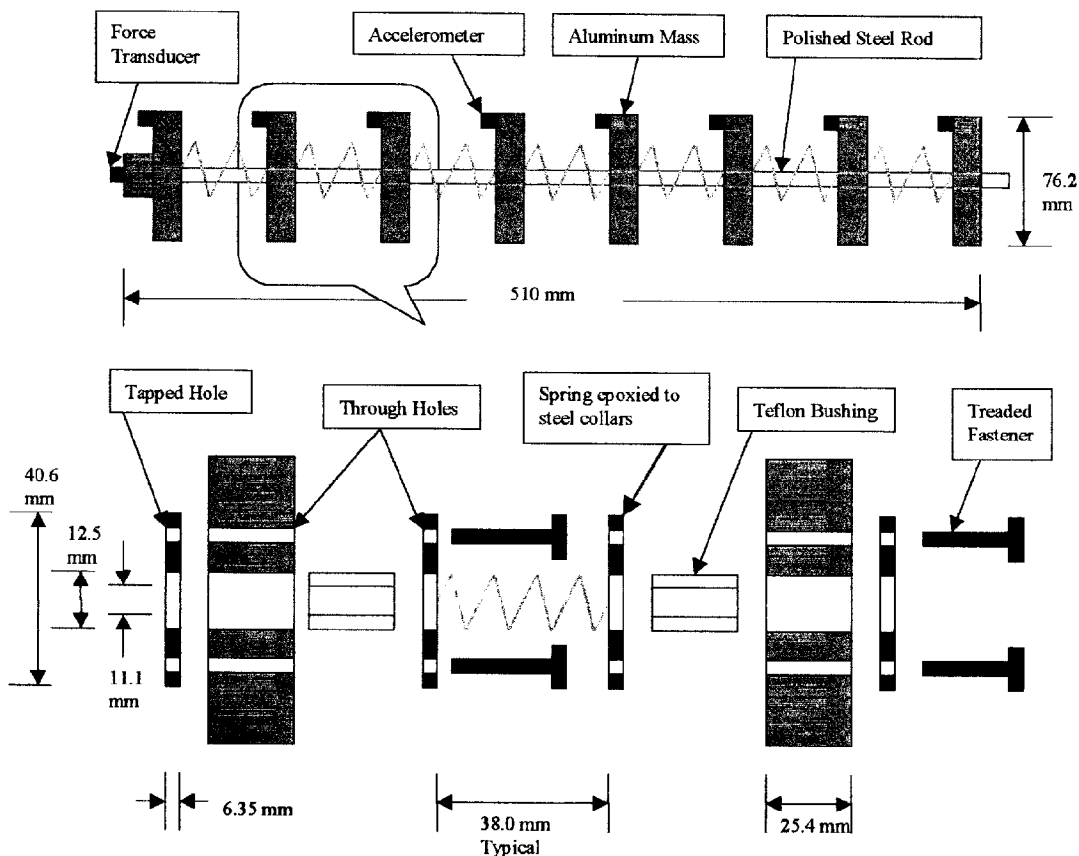


Figure 1: Schematic diagram of the eight degree-of-freedom system.

The undamaged configuration of the system is the state for which all springs are identical and have a linear spring constant. Linear damage in the model is simulated by replacing an original spring with another linear spring which has a spring constant less than that of the original. The replacement spring may be located between any adjacent masses, and thus simulate different locations of damage. The replacement spring may have different degrees of stiffness reduction to simulate different levels of damage.

The nominal values of the system parameters are as follows:

**Mass 1:** 559.3 grams (This mass is located at the end where the shaker is attached or impact-hammer excitation is applied. It is greater than the others because of the hardware needed to attach the shaker.)

**Masses 2 through 8:** 419.4 grams

**Spring constants:**

56.7 kN/m	(322 lb/in)	(undamaged)
43.0 kN/m	(244 lb/in)	(24% stiffness reduction)
49.0 kN/m	(278 lb/in)	(14% stiffness reduction)
52.6 kN/m	(299 lb/in)	( 7% stiffness reduction)

Spring locations are designated by a sequential number with the spring closest to the end of the system where the excitation is applied designated as "No. 1". The "damaged" spring location is given by a number, counting from the excitation end.

Measurements made during damage identification tests are the excitation force applied to mass 1 and the acceleration response of all masses. Excitation is accomplished with either an impact hammer or a 215-N (50 lb) peak force electrodynamic shaker (Figure 2).

Damping in the system is caused primarily by Coulomb friction. Every effort is made to minimize the friction through careful alignment of the masses and springs. A common commercial lubricant, Tri-Flo™, is applied between the Teflon bushings and the support rod. During the conduct of these tests it was found that the excitation force, whether from impulse or random excitation, must be great enough to override the frictional effects.

The data acquisition equipment used in this study was a Hewlett-Packard 3566A data acquisition system. This system is composed of a model 35650 mainframe, 35653A source module, and five 35653A eight-channel input modules (which provided power for the accelerometers and performed an A/D conversion of the transducer signals). A 35651C signal-processing module performs the necessary FFT calculations. A laptop computer was used for data storage and as the platform for the software for controlling the data acquisition system. The force transducer used was a PCB type 204 (nominal sensitivity of 22.5 mv/N), and the accelerometers were Endevco type 2251 A-10 (nominal sensitivity of 10 mv/G). ME'scope™ software was used to determine the required modal properties for each of the configurations tested. Tests performed included the baseline ('undamaged') configuration of the 8-DOF system as well as three levels of damage (7-14-, and 24-percent) sequentially applied at three different locations (springs 1, 5, and 7). Both impact-hammer

excitation and random excitation tests were sequentially performed.

All data used to generate the results presented herein, along with further details of the experiments are available at the Website:

[http://esaea-www.esa.lanl.gov/damage\\_id](http://esaea-www.esa.lanl.gov/damage_id)

### 3 APPLICATION OF THE 'DAMAGE-INDEX' METHOD

#### 3.1 Brief Summary of the Method

The damage index method, developed by Stubbs and Kim [6], locates damage in structures undergoing bending vibration given their characteristic mode shapes measured before and after damage. For structures undergoing bending, it was found that only a few modes are required to obtain reliable results.

The method is based on an examination of modal strain energies in undamaged and damaged beams. It is straightforward to modify the method to account for axial, as opposed to bending, vibrations [5]. Following the derivation for beam bending by Cornwell, et al. [7], the bending strain energy for a Bernoulli-Euler beam is

$$U = \frac{1}{2} \int_0^l EI \left( \frac{d^2 w}{dx^2} \right)^2 dx \quad (1)$$

where EI is flexural rigidity  $l$  denotes beam length,  $w$  is transverse displacement, and  $x$  is the coordinate along the span of the beam. The corresponding energy expression for axial vibrations can be written

$$U = \frac{1}{2} \int_0^l AE \left( \frac{du}{dx} \right)^2 dx \quad (2)$$

where  $u(x)$  denotes the in-plane (axial) displacement field, and AE denotes the axial rigidity.

Making the appropriate modifications to the derivation by Cornwell, et al. [7], it is readily shown that the change in axial rigidity (as opposed to bending rigidity) at the  $k$ th location in the structure for the  $i$ th mode is given by

$$\frac{g_{ik}^*}{g_{ik}} = \frac{\int_{a_k}^{a_{k+1}} \left( \frac{d\psi_i^*}{dx} \right)^2 dx / \int_0^l \left( \frac{d\psi_i}{dx} \right)^2 dx}{\int_{a_k}^{a_{k+1}} \left( \frac{d\psi_i}{dx} \right)^2 dx / \int_0^l \left( \frac{d\psi_i}{dx} \right)^2 dx} \quad (3)$$

where  $\psi_i$  denotes the  $i$ th normal mode shape and  $( )^*$  denotes the case of damage. The above expression is an index of the change in axial rigidity from undamaged to damaged structures.  $a_k \leq x \leq a_{k+1}$  denotes the interval along the span,  $l$ , of the  $k$ th region. If the above equation is reapplied for each sub-region (i.e., element or spring) along the span, a measure of axial rigidity change along the span for the  $i$ th normal mode is produced.

In order to use all measured modes,  $n$ , the damage index,  $\alpha_k$ , for the  $k$ th subregion is defined as

$$\alpha_k = \frac{\sum_{i=1}^n g_{ik}^*}{\sum_{i=1}^n g_{ik}} \quad (4)$$

This method, applied to axial vibrations, is here denoted the 'Modified Damage Index Method'.

### 3.2 Results

Three levels of damage (7-, 14-, and 24-percent stiffness reduction) at three locations (spring 1, spring 5, and spring 7) were sequentially evaluated using the modified damage-index method. As an illustration, results for 24-percent damage at locations 1, 5, and 7 are shown in Figures 3-5, respectively, for impact-hammer excitation. The results of the damage indicator for the first mode, first two modes, first three modes, and first four modes are shown in each case. The following observations can be made for the Modified Damage-Index Method at this significant level of damage:

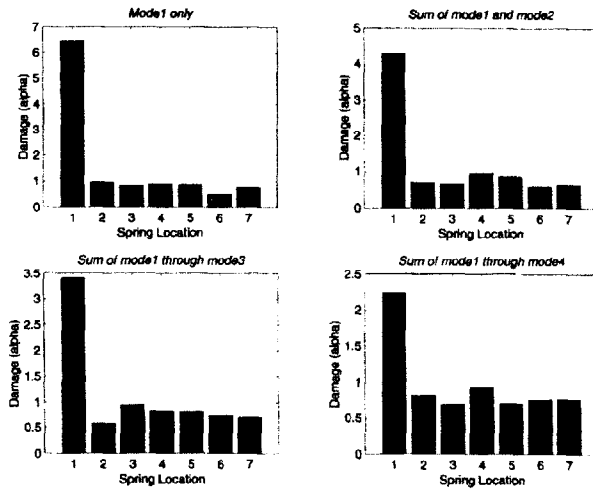


Figure 3: Modified Damage-Index Method for 24-percent Damage at Spring 1, Impulsive Excitation

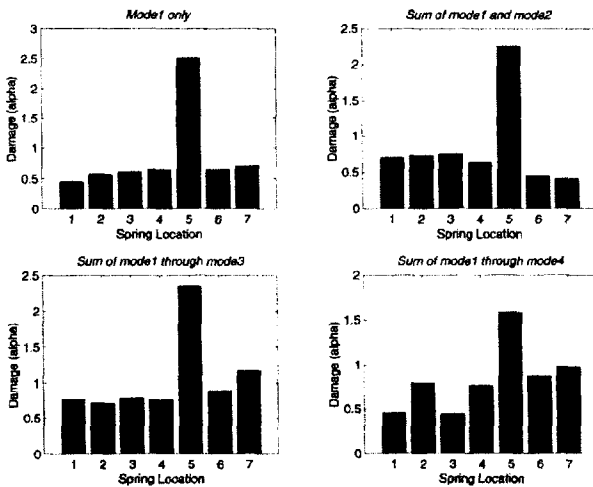


Figure 4: Modified Damage-Index Method for 24-Percent Damage at Spring 5, Impulsive Excitation.

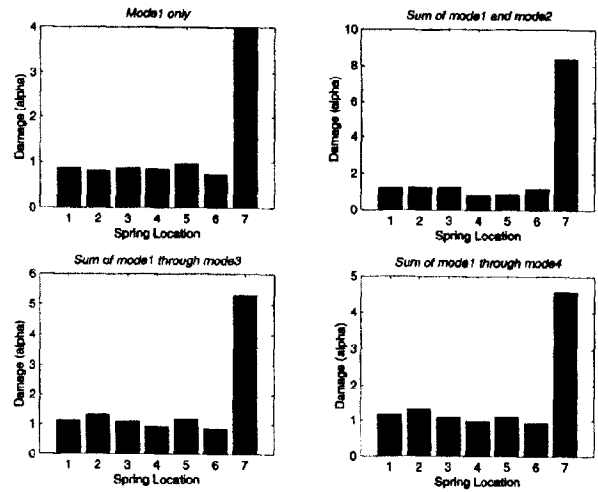


Figure 5: Modified Damage Index Method for 24-Percent Damage at Spring 7, Impulsive Excitation.

1. Damage at the extreme ends of the 8-DOF system (springs 1 and 7) is more readily identified than at an internal location (spring 5).
2. Damage is readily detected using the first mode only. Inclusion of higher modes does not necessarily improve the identification of damage location.

Review of lower (7-percent, 14-percent) levels of damage indicated the following:

1. Damage even at the 7-percent level could readily be detected (e.g., see Figure 6 for 7-percent stiffness reduction of spring 1).

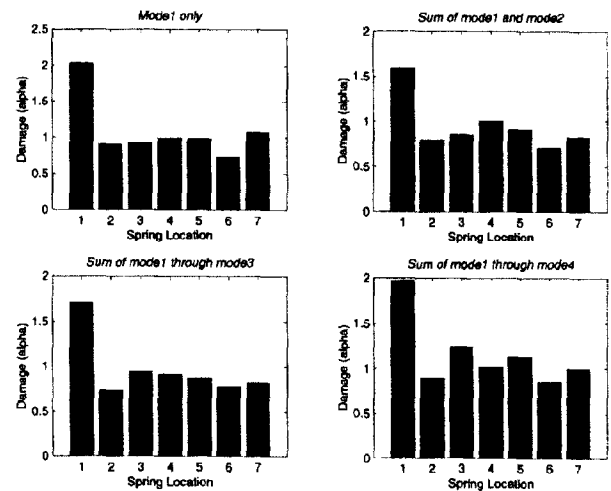


Figure 6: Modified Damage-Index Method for 7-Percent Damage at Spring 1, Impulsive Excitation.

2. Damage at the intermediate (14-percent) level (not shown) was more difficult to detect than damage at lower and higher levels, i.e., the method readily determines the location of damage but does not

provide a direct measure of the level of damage. This anomaly may have been due to friction induced by variations in alignment of the masses during the spring replacement process.

The Modified Damage-Index Method was also applied to random vibration data with similar results.

#### 4 APPLICATION OF THE 'CHANGE-IN-FLEXIBILITY' METHOD

##### 4.1 Brief Summary of Method

As explained in [1,8], the method consists of the following: For the undamaged structure, the flexibility matrix,  $[F]$ , is derived from the modal mass-normalized data as follows:

$$[F^*] = [\Phi][\Omega]^{-1}[\Phi]^T \approx \sum_{i=1}^n \frac{1}{(\omega_i)^2} \{\phi_i\}\{\phi_i\}^T, \quad (5)$$

where  $\{\phi_i\}$  = the  $i$ th mass normalized mode shape,  
 $[\Phi]$  = the mass-normalized mode shape matrix =  $[\phi_1, \phi_2, \dots, \phi_n]$ ,  
 $\omega_i$  = the  $i$ th modal frequency,  
 $[\Omega]$  = The modal stiffness matrix =  $\text{diag. } (\omega_i^2)$ , and  
 $n$  = the number of measured modes.

The approximation in Equation 5 comes from the fact that typically the number of modes identified is less than the number of degrees of freedom needed to accurately represent the motion of the structure. Similarly, for the damaged structure

$$[F^*] = [\Phi^*][\Omega^*]^{-1}[\Phi^*]^T \approx \sum_{i=1}^n \frac{1}{(\omega_i^*)^2} \{\phi_i^*\}\{\phi_i^*\}^T, \quad (6)$$

where the asterisks signify properties of the damaged structure. From the pre- and post- damage flexibility matrices, a measure of the flexibility change caused by the damage can be obtained from the difference of the respective matrices, i.e.,

$$[\Delta F] = [F] - [F^*], \quad (7)$$

where  $\Delta F$  represents the change-in-flexibility matrix. Now, for each column of matrix  $\Delta F$  let  $\bar{\delta}_j$  be the absolute maximum value of the element in the  $j$ th column. Hence,

$$\bar{\delta}_j = \max |\delta_{ij}|, \quad i = 1, \dots, n \quad (8)$$

where  $\bar{\delta}_j$  are elements of matrix  $\Delta F$ .  $\bar{\delta}_j$  is taken to be a measure of the flexibility change at each measurement location. The column of the flexibility matrix corresponding to the largest  $\bar{\delta}_j$  is indicative of the degree of freedom where damage is located.

For the 8-DOF system, damage would actually be located between degrees of freedom. A differencing scheme,

$$\alpha_k = \bar{\delta}_{j+1} - \bar{\delta}_j \quad (9)$$

is then used to determine the particular subregion (spring) which has experienced the damage.

##### 4.2 Results

The method was evaluated using the same data as for the Modified Damage-Index Method (7-, 14-, and 24-percent stiffness reduction at three locations (springs 1, 5, and 7). The following general observations were made:

1. Surprisingly, detection of damage was actually best overall for the case of 7-percent damage, in contrast to results using the Modified Damage-Index Method. Again, variations in alignment of the masses during the spring replacement may have been a factor.
2. Detection of damage was best at the extreme locations (Springs 1 and 7). Damage could only be detected at location 5 for the 7-percent damage level.
3. The ability to detect damage improved as the number of included modes increased, as the flexibility matrix is better approximated with increased numbers of included modes. Recall that for the Modified Damage Index Method, best results were generally obtained using the first mode only.

Results for 7-percent damage are shown in Figures 7 and 8 for damage at locations 1 and 5, respectively. In each of the figures, the four bars at each spring location denote, respectively, inclusion of the first mode only, modes 1 and 2, modes 1-3, and modes 1-4. In this case, damage is clearly indicated for 7-percent damage in spring 1 (See Figure 7) as long as more than one mode is included. For 7-percent damage at spring 5 (Figure 8) damage is indicated in spring 5, but there appear to be false-positive damage indications at springs 2 and 3.

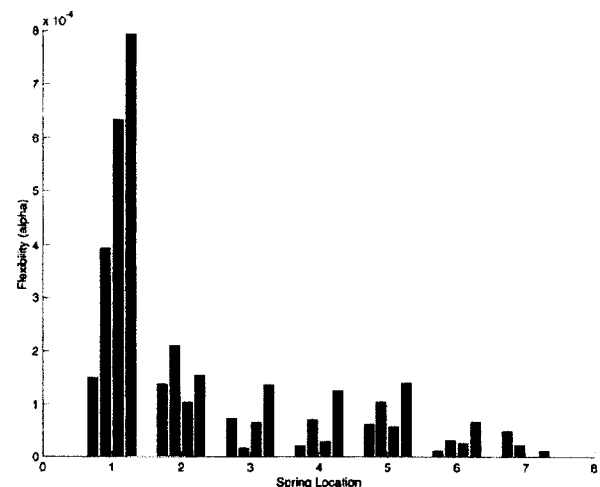


Figure 7: 'Change-in-Flexibility' Method Applied to 7-Percent Damage at Spring 1.

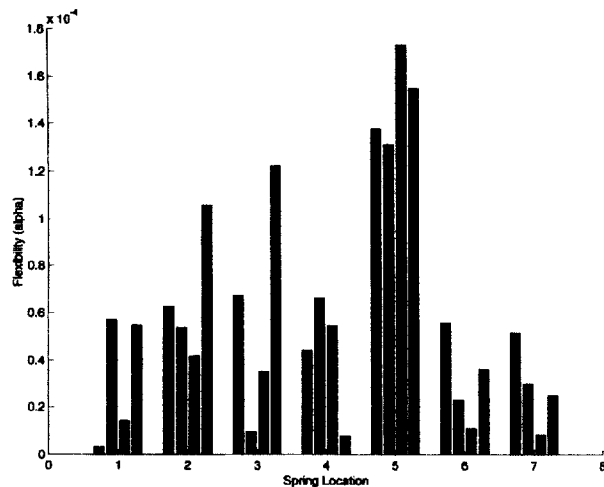


Figure 8: 'Change-in-Flexibility' Method Applied to 7-Percent Damage at Spring 5.

## 5. SUMMARY AND CONCLUSIONS

Two vibration-based, damage detection methods had recently been presented for structures undergoing flexural vibrations. In a recent paper, these methods were applied to structures undergoing axial vibrations. The two methods (Modified 'Damage-Index' and 'Change-in-Flexibility') showed promise based upon purely numerical evaluations.

In this paper, the two methods were evaluated using experimental data from an 8-DOF spring-mass system. A physical spring-mass model was constructed. Experiments were then performed and the modal properties of the system were determined in "undamaged" and "damaged" states. Excitation was provided either by an impact hammer or an electro-dynamic shaker. "Damage" was introduced by replacing one of the springs (1, 5, or 7) by a spring of lower stiffness (stiffness reduction of 7-, 14-, or 24-percent).

The Modified Damage-Index method functioned well in locating damage at all three levels of damage investigated. The method, however, performed better at isolating damage at (exterior) spring locations 1 and 7 than for the interior spring location 5. Damage was successfully located using the first mode only.

Results were not as favorable with the 'Change-in-Flexibility' Method. Although damage location could be detected for 7-percent damage at each location, results surprisingly deteriorated at 14-percent damage. At the 24-percent level, damage could easily be located at spring locations 1 and 7, but not at location 5. Overall, results using this method were somewhat unreliable, depending strongly upon damage location, damage level, and number of modes included.

Future work (in progress) involves detection of nonlinear damage. The above-described 8-DOF system has been modified to simulate nonlinear damage by placing rods (impactors) on one mass that limit the amount of motion that

the mass may move relative to an adjacent mass. The degree of damage is controlled by changing the amount of relative motion permitted before contact. Methods of statistical pattern recognition (not dependent on modal analysis) are being applied to identify the presence of this nonlinear damage.

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